ASSUME TISSUE WITH COLLAGEN FIBERS ORIENTED ALONG

$$\underset{\sim}{c} = \frac{2}{3}\underset{\sim}{e_1} + \frac{2}{3}\underset{\sim}{e_2} + \frac{1}{3}\underset{\sim}{e_3}$$ IS SUBJECT

TO THE DEFORMATION

$$[\underset{\sim}{\epsilon}] = \begin{bmatrix} 1.5 & 0 & 0 \\ 0 & -0.5 & 0 \\ 0 & 0 & -0.5 \end{bmatrix}$$

A) IS THIS TISSUE INCOMPRESSIBLE?

Assume undeformed dimensions:

L_1, L_2, L_3

And deformed: l_1, l_2, l_3

$$\epsilon_{11} = \frac{l_1 - L_1}{L_1} = 1.5$$

$$l_1 = 2.5L_1$$

SIMILARLY,

$$l_2 = 0.5L_2$$

$$l_3 = 0.5L_3$$

$$V = L_1 L_2 L_3$$

$$v - V = l_1 l_2 l_3 - L_1 L_2 L_3 \neq 0$$

B) WHAT IS THE LENGTH OF A FIBER THAT IS INITALLY 1μm LONG AFTER THE DEFORMATION?

Δ in length $\Rightarrow$ normal strain along $\underset{\sim}{c}$ $\Rightarrow$

$$\underset{\sim}{c} \cdot \underset{\approx}{\epsilon} \cdot \underset{\sim}{c} = \frac{l_f - L_f}{L_f}$$

$L_f \rightarrow$ fiber length ($L_f = 1\mu m$)

$$c_i \underset{\sim}{e_i} \cdot \epsilon_{jk} \underset{\sim}{e_j} \otimes \underset{\sim}{e_k} \cdot c_l \underset{\sim}{e_l}$$

$$= c_i \epsilon_{jk} c_l (\underset{\sim}{e_i} \cdot \underset{\sim}{e_j})(\underset{\sim}{e_k} \cdot \underset{\sim}{e_l})$$

$$= c_i \epsilon_{jk} c_l \delta_{ij} \delta_{kl}$$

$$= c_i \epsilon_{ij} c_j$$

b/c ϵ_{ij} is a diagonal matrix, this sum is

$$c_1^2 \epsilon_{11} + c_2^2 \epsilon_{22} + c_3^2 \epsilon_{33}$$

$$\frac{4}{9}(1.5) + \frac{4}{9}(-0.5) + \frac{1}{9}(-0.5)$$

$$\frac{6}{9} - \frac{2}{9} - \frac{0.5}{9} = \frac{3.5}{9} = 0.3888$$

$$\frac{l_f - L_f}{L_f} = 0.3888 = l_f \; 1.388 \, \cancel{\mu}m$$